1. Find $\begin{vmatrix} 8 & 6 & 7 \\ 5 & 3 & 0 \\ 9 & 1 & 8 \end{vmatrix}$.

   a. -202 b. 3 c. 106 d. 278 e. NOTA

2. Find the sum of the elements in the third row of $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 4 & 0 & 4 \end{bmatrix}^{-1}$

   a. $-\frac{7}{6}$ b. $-\frac{1}{4}$ c. $-\frac{1}{6}$ d. $\frac{1}{2}$ e. NOTA

3. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \prod_{k=1}^{10} \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$. Find $a + b + c + d$.

   a. $10! + 2$ b. 7 c. 57 d. 40 e. NOTA

4. Let the eigenvalues of $\begin{bmatrix} 6 & 2 \\ 9 & -1 \end{bmatrix}$ be λ_1 and λ_2. Find $|\lambda_1 - \lambda_2|$.

   a. 5 b. 7 c. 9 d. 11 e. NOTA

5. If $A = \begin{bmatrix} 0 & 5 & 7 \\ 4 & 3 & 1 \\ 0 & 2 & 3 \end{bmatrix}$, find $\det(A^3)$.

   a. -64 b. -33 c. 0 d. 33 e. NOTA

6. Emily applies Cramer's rule to find the solution (x, y, z) to a system of linear equations with three variables. She determines that $y = \frac{\begin{vmatrix} 2 & -9 & -3 \\ 3 & 4 & -4 \\ 5 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 & -3 \\ 3 & -2 & -4 \\ 5 & 8 & -1 \end{vmatrix}}$. Find $x + z$.

   a. 1 b. 7 c. 10 d. 12.5 e. NOTA

7. Let $M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Find $M^{10} - M^9 - M^8$.

   a. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$ e. NOTA

8. Let $k = \begin{vmatrix} A & B & C \\ D & E & F \\ G & H & I \end{vmatrix}$. Find $\begin{vmatrix} 2A & 2B & 2C \\ G & H & I \\ D + 3A & E + 3B & F + 3C \end{vmatrix}$.

   a. $-2k$ b. $2k + 3$ c. $6k$ d. $-6k$ e. NOTA

9. Find the second entry in the first row of the classical adjoint matrix of $\begin{bmatrix} 5 & 2 & -1 \\ 6 & -3 & 9 \\ 4 & -3 & 7 \end{bmatrix}$.

a. -17 b. -6 c. 6 d. 11 e. NOTA

10. Mike is asked to show that the matrix $M = \begin{bmatrix} 14 & 30 \\ -6 & -13 \end{bmatrix}$ is diagonalizable. He does this by finding an invertible matrix P and a diagonal matrix A that satisfy $M = P \times A \times P^{-1}$. Mike finds $P = \begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$. What diagonal matrix A will satisfy $M = P \times A \times P^{-1}$?

a. $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 3 & 0 \\ 0 & 42 \end{bmatrix}$ d. $\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ e. NOTA

11. In an effort to one-up Mike, Matthew takes Mike's matrix $M = \begin{bmatrix} 14 & 30 \\ -6 & -13 \end{bmatrix}$ from the previous problem and computes M^{10}. Now it's your turn. Find M^{10}.

a. $\begin{bmatrix} 5040 & 24793 \\ -208 & -1023 \end{bmatrix}$ b. $\begin{bmatrix} 5500 & 3888 \\ -1478 & -255 \end{bmatrix}$ c. $\begin{bmatrix} -5040 & -14642 \\ 1022 & 2047 \end{bmatrix}$
d. $\begin{bmatrix} 5116 & 10230 \\ -2046 & -4091 \end{bmatrix}$ e. NOTA

12. Evaluate $\sum_{k=1}^{3} \begin{bmatrix} 1 & k & k^2 \\ k & k^2 & k^3 \\ k^2 & k^3 & k^4 \end{bmatrix}$

a. $\begin{bmatrix} 2 & 5 & 13 \\ 5 & 13 & 35 \\ 13 & 35 & 97 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 6 & 12 \\ 6 & 12 & 24 \\ 12 & 24 & 48 \end{bmatrix}$ c. $\begin{bmatrix} 2 & 3 & 5 \\ 3 & 5 & 9 \\ 5 & 9 & 17 \end{bmatrix}$ d. $\begin{bmatrix} 3 & 6 & 14 \\ 6 & 14 & 36 \\ 14 & 36 & 98 \end{bmatrix}$ e. NOTA

13. What value of x makes $\begin{bmatrix} 2 & -2 & -4 \\ -1 & x & 4 \\ 1 & -2 & -3 \end{bmatrix}$ idempotent?

a. 1 b. 2 c. 3 d. 4 e. NOTA

14. Jeffrey and Eddie have found a way to send secret messages to each other. First, they write a message in a matrix. Then, they convert the letters in their message into numbers. A turns into 0, B to 1, C to 2,..., and Z to 25. Finally, they left multiply their message matrix by $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$. So, they would encrypt the message $\begin{bmatrix} B & A & D \\ D & O & G \end{bmatrix}$ as $\begin{bmatrix} 9 & 28 & 21 \\ 5 & 14 & 12 \end{bmatrix}$ since $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 3 & 14 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 28 & 21 \\ 5 & 14 & 12 \end{bmatrix}$. If Jeffrey sends Eddie the encrypted message $\begin{bmatrix} 47 & 42 & 101 & 20 \\ 29 & 28 & 61 & 12 \end{bmatrix}$, what is his message about?

a. Math b. Food c. Emotions d. Superheroes e. NOTA

15. Which of the following are eigenvectors of $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$?

$$I)\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} \qquad II)\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad III)\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

a. I only b. II only c. I and III only d. II and III only e. NOTA

16. Find $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 4 & 0 & 4 \end{vmatrix}$

a. 0 b. 10 c. 48 d. 100 e. NOTA

17. Which represents the set of solutions $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$ to the following system of equations?

$$\begin{cases} w + x - 2z = 2 \\ x + 3z = 23 \\ y - 4z = -20 \end{cases}$$

a. $\left\{\begin{bmatrix} -1 \\ 11 \\ -4 \\ -4 \end{bmatrix} + t\begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix} \middle| t \in \mathbb{R}\right\}$ b. $\left\{\begin{bmatrix} 9 \\ 5 \\ 4 \\ 6 \end{bmatrix} + t\begin{bmatrix} -2 \\ 5 \\ 1 \\ 2 \end{bmatrix} \middle| t \in \mathbb{R}\right\}$ c. $\left\{\begin{bmatrix} 7 \\ 8 \\ 3 \\ 5 \end{bmatrix} + t\begin{bmatrix} 2 \\ -3 \\ 1 \\ 1 \end{bmatrix} \middle| t \in \mathbb{R}\right\}$

d. $\left\{\begin{bmatrix} -21 \\ 23 \\ -20 \\ 0 \end{bmatrix} + t\begin{bmatrix} 5 \\ -3 \\ 4 \\ 1 \end{bmatrix} \middle| t \in \mathbb{R}\right\}$ e. NOTA

18. Find the sum of the elements in $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}^6$.

a. -1 b. 1 c. 2 d. 3 e. NOTA

19. To bake a cake, Georgina uses 3 cups of flour and 2 eggs. To make a batch of brownies, Georgina uses 2 cups of flour and 4 eggs. Georgina has 21 cups of flour and 22 eggs, and she wishes to use all of her ingredients. If she is to make C cakes and B batches of brownies, which matrix multiplication will help Georgina determine $\begin{bmatrix} C \\ B \end{bmatrix}$?

a. $\begin{bmatrix} \frac{3}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 21 \\ 22 \end{bmatrix}$ b. $\begin{bmatrix} \frac{-3}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{-1}{2} \end{bmatrix}\begin{bmatrix} 21 \\ 22 \end{bmatrix}$ c. $\begin{bmatrix} \frac{3}{8} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 21 \\ 22 \end{bmatrix}$ d. $\begin{bmatrix} \frac{1}{2} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 21 \\ 22 \end{bmatrix}$ e. NOTA

20. Georgina really enjoyed the food in the previous problem, so she has begun to think about how many cakes and brownie batches she would make if her ingredient amounts were different. If she had 14 cups of flour and 20 eggs, she would make W cakes (and some number of brownies). If she had 25 cups of flour and 30 eggs, she would make X cakes (and some number of brownies). With 11 cups of flour and 18 eggs, she would make Y cakes (and some number of brownies). If she had 18 cups of flour and 20 eggs, she would make Z cakes (and some number of brownies). Find $W+X+Y+Z$.

a. 10 b. 12 c. 16 d. 20 e. NOTA

21. Given $Q^{-1}=\begin{bmatrix}4 & -7\\ -1 & 3\end{bmatrix}$, $R^{-1}=\begin{bmatrix}2 & -5\\ 3 & -8\end{bmatrix}$, and $S^{-1}=\begin{bmatrix}1 & -1\\ 2 & -3\end{bmatrix}$, find $(QRS)^{-1}$.

a. $\begin{bmatrix}-7 & 16\\ -34 & 77\end{bmatrix}$ b. $\begin{bmatrix}54 & -95\\ -31 & 50\end{bmatrix}$ c. $\begin{bmatrix}-20 & 55\\ -47 & 129\end{bmatrix}$ d. $\begin{bmatrix}-45 & 95\\ -73 & 154\end{bmatrix}$ e. NOTA

22. A set of vectors is called "linearly dependent" if one of its elements can be expressed as a linear combination of the other elements. For instance $\left\{\begin{bmatrix}1\\3\\-2\end{bmatrix}, \begin{bmatrix}5\\1\\1\end{bmatrix}, \begin{bmatrix}11\\5\\0\end{bmatrix}\right\}$ is linearly dependent since the third vector can be expressed as the first vector plus twice the second. Find the sum of all values of x for which $\begin{bmatrix}x\\-1\\1\end{bmatrix}$, $\begin{bmatrix}7\\x\\5\end{bmatrix}$, and $\begin{bmatrix}3\\x\\2\end{bmatrix}$ form a linearly dependent set.

a. $-\frac{2}{3}$ b. 0 c. 1 d. $\frac{4}{3}$ e. NOTA

23. On the island of Sperry, a very odd spell has been cast. Every night, at midnight, 40% of the island's humans turn into tortoises and $\frac{1}{3}$ of the island's tortoise's turn into humans. After living under this spell for a long enough time, the island's human population H and tortoise population T have reached an equilibrium. That is, H and T are constant. $\frac{H}{T}$ can be expressed as $\frac{a}{b}$ where a and b are relatively prime integers. Find $a+2b$. Assume no births or deaths occur on the island of Sperry.

a. 15 b. 16 c. 17 d. 18 e. NOTA

24. Let $M=\begin{bmatrix}5 & 6\\ 3 & 4\end{bmatrix}$. Find M^{-1}.

a. $\begin{bmatrix}4 & -6\\ -3 & 5\end{bmatrix}$ b. $\begin{bmatrix}-5 & 3\\ 6 & -4\end{bmatrix}$ c. $\begin{bmatrix}8 & -12\\ -6 & 10\end{bmatrix}$ d. $\begin{bmatrix}-10 & 6\\ 12 & -8\end{bmatrix}$ e. NOTA

25. Find $\begin{vmatrix}5 & 6\\ 6 & 7\end{vmatrix}$.

a. -11 b. -1 c. 1 d. 9 e. NOTA

26. Find $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 1 \\ 1 & 3 & 6 & 10 & 4 & 1 \\ 1 & 4 & 10 & 6 & 3 & 1 \\ 1 & 5 & 4 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}$.

a. -65 b. -32 c. 33 d. 64 e. NOTA

27. Let $M = \begin{bmatrix} 5 & 7 & -3 \\ -2 & 1 & 6 \\ 8 & 5 & 5 \end{bmatrix}$ and let C be the cofactor matrix of M (that is, each (i, j) entry in C should be the (i, j) cofactor of M). If $A = C^{\mathrm{T}}$, find the sum of all the entries in $\frac{1}{\det(M)} MA$.

a. 0 b. 1 c. 3 d. 9 e. NOTA

28. Let A be a 2×2 matrix for which $A\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ and $A\begin{bmatrix} -1 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find the sum of the entries in A^{-1}.

a. 2 b. 4 c. 8 d. 16 e. NOTA

29. Which of the following are true?

I) For square matrices A and B of equal dimension, $(A + B)^2 = A^2 + 2AB + B^2$ if and only if A and B commute.

II) $\text{adj}(AB) = \text{adj}(A) \times \text{adj}(B)$ for all square matrices A and B of equal dimension.

III) All nilpotent matrices are singular.

a. I only b. III only c. I and II only d. I, II, and III e. NOTA

30. Find the product $\begin{bmatrix} 1 & 6 & -4 \\ 7 & 2 & 3 \\ 2 & -5 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 3 \\ -6 & 1 & 1 \end{bmatrix}$.

a. $\begin{bmatrix} 2 & 25 & 17 \\ -12 & 6 & 30 \\ 18 & -26 & -8 \end{bmatrix}$ b. $\begin{bmatrix} 1 & -5 & -8 \\ 37 & -29 & 34 \\ 3 & -39 & 28 \end{bmatrix}$ c. $\begin{bmatrix} 50 & 35 & 17 \\ 4 & 14 & 30 \\ 18 & 26 & 22 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 33 & -19 \\ 40 & 0 & 18 \\ 22 & 26 & -20 \end{bmatrix}$ e. NOTA